

Subsets

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Definition

Suppose that A and B are sets.

- ▶ If every element of A is also an element of B , then we say that A is a **subset** of B . This can be written using the subset symbol $A \subseteq B$.
- ▶ If at least one element of A is not an element of B , then A is not a subset of B . This can be written $A \not\subseteq B$.

$$\mathbb{Z} \subseteq \mathbb{Q}$$

distinguish $\subseteq$
from $\subset$

$$\mathbb{Q} \not\subseteq \mathbb{Z} \quad \cancel{\mathbb{Z} \not\subseteq \mathbb{Q}} \quad \text{Note } \mathbb{Z} \not\subseteq \mathbb{Q}$$

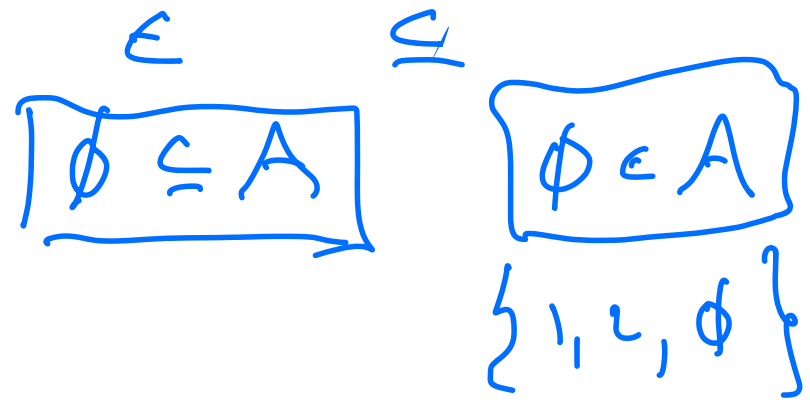
$$\{1, 2, 4, 6, 8, 10, 12, \dots\} \not\subseteq \text{set of even numbers}$$

$$\begin{aligned} \text{For any set } A, \quad A &\subseteq A \\ \text{For any set } A, \quad \emptyset &\subseteq A \end{aligned}$$

$$A \not\subseteq B \quad A \subset B$$

Example

► $\{2, 3, 7\} \subseteq \{2, 3, 4, 5, 6, 7\}$



► $\{2, 3, 11\} \not\subseteq \{2, 3, 4, 5, 6, 7\}$

Example

► $\mathbb{N} \subseteq \mathbb{Z}$

1, 2, 3,, -3, -2, ..., -1, 0, 1, 2, ...

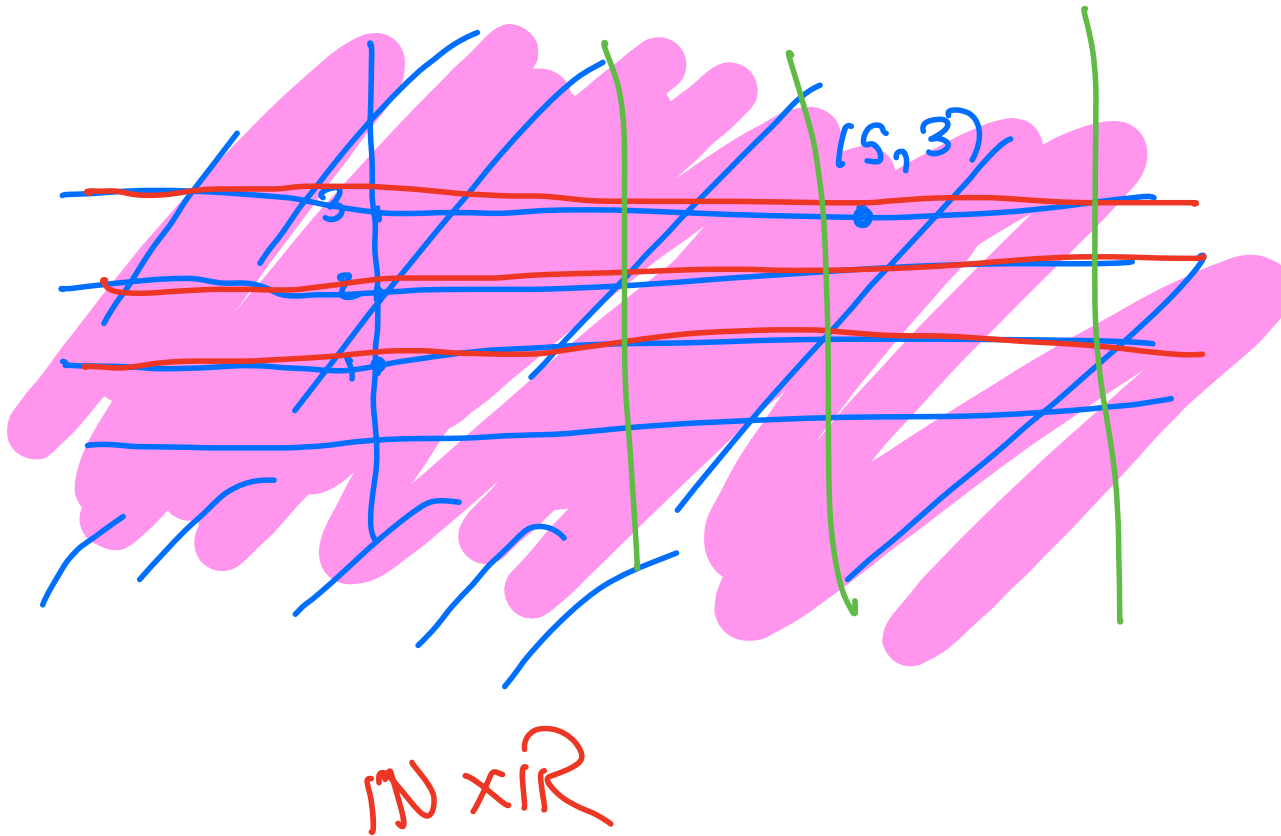
$$\mathbb{N} = \{x \in \mathbb{Z} \mid x \geq 1\}$$

► $\mathbb{Z} \subseteq \mathbb{Q}$

Example

► $\mathbb{R} \times \mathbb{N} \subseteq \mathbb{R} \times \mathbb{R}$

● $\mathbb{R} \times \mathbb{N} = \left\{ (r, n) \mid r \in \mathbb{R} \quad n \in \mathbb{N} \right\}$



Example

► $\mathbb{N} \times \mathbb{R} \not\subseteq \mathbb{R} \times \mathbb{N}$

Example

- ▶ For any set A , $A \subseteq A$.

The Empty Set

- ▶ The empty set is a subset of every set.